RELATIVE NAVIGATION STRATEGIES
FOR THE MAGNETOSPHERIC MULTISCALE MISSION

Cheryl Gramling[1], Russell Carpenter[2], Taesul Lee[3], Anne Long[4]

[1]NASA Goddard Space Flight Center, Greenbelt MD 20771 USA, Email: Cheryl.J.Gramling@nasa.gov
[2] NASA Goddard Space Flight Center, Greenbelt MD 20771 USA, Email: Russell.Carpenter@nasa.gov
[3]a.i. solutions, Inc, 10001 Derekwood Lane, Lanham MD 20706 USA, Email: lee@ai-solutions.com
[4]a.i. solutions, Inc, 10001 Derekwood Lane, Lanham MD 20706 USA, Email: long@ai-solutions.com

ABSTRACT

This paper evaluates several navigation approaches for the Magnetospheric Multiscale (MMS) mission, which consists of a tetrahedral formation of satellites flying in highly eccentric Earth orbits. For this investigation, inter-satellite separations of approximately 10 kilometers near apogee are used for the first two phases of the MMS mission. Navigation approaches were studied using ground station two-way Doppler measurements, Global Positioning System (GPS) pseudorange measurements, and cross-link range measurements between the members of the formation. An absolute position accuracy of 15 kilometers or better can be achieved with most of the approaches studied, and a relative position accuracy of 100 meters or better can be achieved at apogee in several cases.

1. INTRODUCTION

Autonomous formation flying will enable many small, inexpensive satellites to gather concurrent science data. The Mission Engineering and Systems Analysis (MESA) Division at Goddard Space Flight Center (GSFC) is developing advanced spacecraft systems to provide autonomous navigation and control of formation flyers. To support this effort, MESA is assessing the relative navigation accuracy achievable for proposed formations using ground station (GS), Global Positioning System (GPS), and cross-link measurements. This paper evaluates the performance of several relative navigation algorithms for Phases 1 and 2 of the Magnetospheric Multiscale (MMS) mission.

The baseline MMS mission consists of two equatorial orbit phases (Phases 1 and 2), one double lunar swingby (DLS) phase, and one polar orbit phase. The Phase 1 orbits are approximately 1.2 x 12 Earth radii at an inclination of 10 degrees. At the end of Phase 1, maneuvers will be performed to raise the apogee to 30 Earth radii for Phase 2. For this study, inter-satellite separations near apogee of about 10 kilometers (km) are used for both Phases 1 and 2. Inter-satellite separations near perigee are approximately 170 and 65 km for Phases 1 and 2, respectively.

Filtered solutions, which are computed using the Global Positioning System (GPS) Enhanced Onboard Navigation System (GEONS) [1], are analyzed to identify navigation approaches that will provide the navigation accuracy needed to meet the science objectives for the MMS mission. The science accuracy requirements are (1) a post-processing knowledge of the satellite position to within 100 km and (2) knowledge of the inter-satellite distances to within 1 percent of the actual separation (e.g., 100 meters (m) near apogee for 10-km separations). The more stringent navigation accuracy requirements associated with satisfying the MMS Phase 1 formation control objectives are addressed to some extent in this paper and will be evaluated further in a follow-on study.

The baseline orbit determination (OD) approach for MMS uses a ground-based system to process two-way GS Doppler measurements provided by a ground tracking network. In addition, cross-link range measurements will be available from an Inter-spacecraft Ranging and Alarm System (IRAS). Processing tracking data acquired onboard, such as cross-link range data, in the ground-based solutions will require downlinking of these measurements to the ground system. If real-time or near real-time OD is needed, the recommended approach is to perform OD onboard using GPS pseudorange (PR) measurements or a combination of GPS PR and cross-link range measurements.

Section 2 presents the simulation characteristics. Sections 3 and 4 discuss the MMS Phase 1 and Phase 2 navigation analysis results, respectively. Section 5 summarizes the conclusions.

1. SIMULATION CHARACTERISTICS

High-fidelity simulations were performed to compare the absolute and relative accuracies using GPS PR measurements, cross-link range measurements, GS range and Doppler measurements, and combinations of these measurements. The truth trajectories used in the measurement data simulation were generated using the Goddard Trajectory Determination System (GTDS) with a 50x50 Joint Gravity Model 2, solar radiation

pressure, atmospheric drag, and point-mass gravity due to the Sun, Moon, Mars, Venus, Saturn, and Jupiter [2]. Table 1 lists the measurement rate and errors applied in the simulation.

Table 1. Measurement Simulation Parameters

Parameter	Nominal Values
GPS Pseudorange:	
• Measurement rate	every 60 seconds for up to 12 visible GPS SVs
• Random noise (1-σ)	2 m above 40 dB-Hz 10 m between 28 and 40 dB-Hz
• GPS Ephemeris Error (1-σ)	2 m
• Receiver clock stability	Oven-stabilized oscillator
− over one second	1 part in 10^{11}
− over one day	2 parts in 10^{11}
− Allan variance parameters	h_0 (seconds)= 8×10^{-28} h_{-2} (/second)=2.4×10^{-22}
Cross-link Range:	
• Measurement rate	every 5 minutes for all cross-link (local-to-remote and remote-to-remote)
• Random noise (1-σ)	10 m for separations <30 km 0.033% of separation for separations >30 km
GS Two-Way:	
• Measurement rate	every 10 seconds; three 20-minute contacts per day from Wallops Is., Hawaii, and Madrid
• Random noise (1-σ) Range Doppler	2 m 35 mHz
Atmospheric Delays:	
• Ionospheric and tropospheric effects for GS measurements	Based on [2] with 15-degree elevation angle mask
• Ionospheric effects for GPS pseudorange	Signals included for HORP >500 km for Phase 1 and >1000 km for Phase 2

GPS PR measurements were simulated using a GPS receiver signal acquisition threshold of 28 dB-Hertz (dB-Hz). To reduce the ionospheric delay effects on the measurements, a minimum height of the ray path (HORP) of 500 km was used for Phase 1 and 1000 km for Phase 2. Fig. 1 shows the GPS Space Vehicle (SV) visibility for a satellite in MMS Phase 1 and Phase 2 orbits using these signal and geometric constraints. The orbital periods of the Phase 1 and Phase 2 orbits are approximately 1 day and 3.6 days, respectively. For the Phase 1 orbits, there is about a 2-hour gap in GPS visibility around apogee. For Phase 2, the gap increases to about 2.6 days around apogee. The GS contacts are evenly distributed around the orbits, with at least a 30-minute gap between contacts. The estimated state vector consisted of the absolute state (position, velocity, and GPS receiver clock bias and drift) of the local satellite and optionally the relative states for the remote satellites.
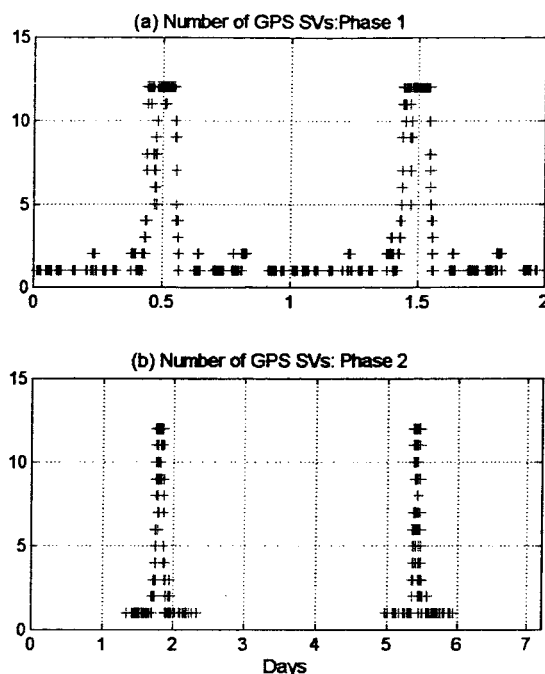

Fig. 1. GPS Space Vehicle Visibility

1. SUMMARY OF MMS PHASE 1 NAVIGATION SOLUTIONS

Reference [3] presents MMS Phase 1 analysis results from earlier studies using GPS PR, cross-link range, GS two-way range and Doppler measurements, and combinations of these measurement types. These studies demonstrated that including cross-link range measurements in the solution significantly reduces the relative errors. In the case of GS measurements, the relative OD accuracy requirements can be met only when cross-link range measurements are included in the solution. This section describes the results of three additional navigation solutions for MMS Phase 1: (1) Monte Carlo multiple-satellite solutions using GPS PR and cross-link range measurements, (2) single-satellite solutions for the local satellite using GPS PR on the local satellite and cross-link range measurements from the remote to local satellite, and (3) multiple-satellite solutions including formation maintenance maneuvers.

1.1 Monte Carlo Simulations Using GPS PR and Cross-link Range Measurements

Monte Carlo simulations were performed for a 7-day definitive and 7-day predictive arc. Solutions were obtained using GPS PR and cross-link range measurements simulated using the measurement noise and user clock stability parameters listed in Table 1. Solutions obtained using cross-link range measurements combined with other types of measurements were found to be sensitive to the relative data weights and the time

of the cross-link data introduction. In general, cross-link range measurements should be introduced after the filter reaches initial convergence. Once introduced and processed, cross-link range data keep the estimates for the satellites in the formation at relative distances consistent with the accuracy of measured cross-link range. In the resulting solutions, the position errors are evenly distributed among all satellites, thereby reducing the relative errors. For the solutions included in this Monte Carlo simulation, the cross-link range measurements were introduced after the first perigee (approximately 12 hours from the solution epoch). The measurement standard deviations (SD) for GPS PR and cross-link range measurements were 25 m and 500 m, respectively.

Table 2 lists the steady-state definitive root-mean-square (RMS) and maximum (Max) errors and 7-day predictive maximum errors for Satellite 4. Other satellites have similar error statistics. These errors are time-wise ensemble statistics of 26 solutions, obtained by varying random number seeds of all measurement-related random errors including the GPS receiver clock errors. The relative errors quoted are those for Satellite 4 with respect to Satellite 1. The maximum errors occur prior to perigee just before the start of good GPS SV visibility.

Table 2. Monte Carlo Simulation Results for Satellite 4

Estimate Errors	Steady State RMS	Steady State Max	7-day Predictive Max
Absolute Position (m)	55	276	8800
Absolute Velocity (m/s)	0.003	0.022	6.2
Absolute SMA (m)	22	102	200
Relative Position (m)	2	7	80
Relative Velocity (m/s)	0.0002	0.002	0.070
Relative SMA (m)	0.5	8.0	8.9

The definitive absolute and relative position errors are well below the MMS requirements of 100 km absolute error and 1% of the separation relative error. Seven-day predictive relative errors are also well below 7% of the relative separation, a derived prediction accuracy requirement associated with formation control.

1.1 Single Satellite Solutions Using GPS PR and Cross-link Measurements

In this scenario, the absolute states of all remote satellites are estimated individually using GPS PR measurements. The local satellite receives the estimated states of the remote satellites along with the cross-link range measurements from the remote satellites. Two approaches for computing the local satellite state are compared: (1) Solution 1 computed using only GPS PR measurements and (2) Solution 2 computed using GPS PR and cross-link measurements from the remote to local satellite.

Table 3 summarizes the steady-state relative position errors between the local satellite (1) and the remote satellites (2, 3, 4) from Solutions 1 and 2. The relative errors obtained using Solution 2 are at least 16% smaller than those obtained using Solution 1. The maximum errors occur near perigee; the errors near apogee are below 75 m. Although these results meet the MMS definitive accuracy requirements, they are substantially larger than the Monte Carlo simulation results given in the previous section. The main reason for this difference is that the Monte Carlo solutions are simultaneous four-satellite solutions using all cross-link range measurements (i.e., remote-to-remote as well as remote-to-local).

Table 3. Relative Steady-State Position Errors Based on Single Satellite Solutions

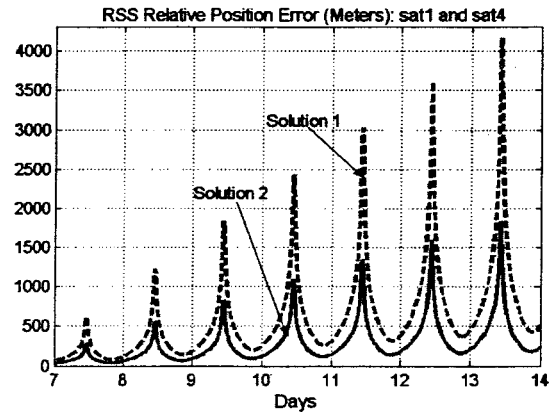| Satellite Pair	Relative Position Error (m)			
	Solution 1		Solution 2	
	RMS	Max	RMS	Max
Satellites 1 and 2	31	120	26	104
Satellites 1 and 3	37	167	24	122
Satellites 1 and 4	39	148	27	126

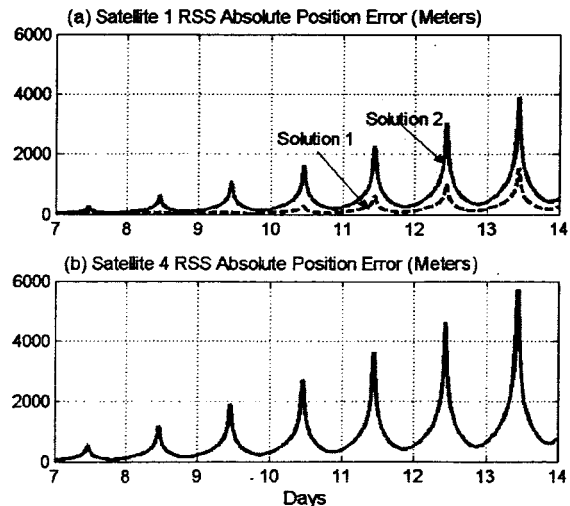Fig. 2. Relative Position Prediction Errors: Single Satellite Solutions

Fig. 3. Absolute Position Prediction Errors: Single Satellite Solutions

Fig. 2 shows the predicted relative errors for Satellite 4 with respect to Satellite 1 from Solutions 1 and 2. The relative errors grow much faster in Solution 1 than in Solution 2. Fig. 3 shows the corresponding absolute errors. The absolute position errors of Solution 2 are significantly larger than those of Solution 1. This brings Solution 2 of Satellite 1 closer to the Satellite 4 solution in terms of the absolute position errors. Therefore, when using Solution 2, the relative position errors between Satellites 1 and 4 become smaller.

1.2 Formation Maintenance Solutions Using GPS PR and Cross-link Range Measurements

Maneuvers must be performed to maintain the satellites in the desired formation. A preliminary assessment was made of the effect of formation maintenance maneuvers on the filtered solutions. Preliminary maneuver data (impulsive delta-V's), which were provided by the MESA maneuver design team, include five maneuvers for each satellite, which are applied approximately at the times with true anomalies of 180, 90, 0, 90, and 180 degrees during the fourth orbital period. The magnitudes of these maneuvers are smaller than expected for the formation maintenance maneuvers, the largest one being 1.43 centimeters per second. GTDS truth ephemerides were generated including these maneuvers and used to simulate GPS PR and cross-link range measurements. Since the maneuvers are all small, they were not modeled in the GEONS state propagation. Instead, the GEONS filter was commanded to increase the state covariance near the time of each maneuver. The three maneuvers in the middle, which are applied over a total time span of about 1 hour around the perigee, have the largest delta-V's.

Four multiple-satellite GEONS filter solutions were computed: (1) Solution 1 without maneuvers, (2) Solution 2 in which maneuvers were included in the data simulations but not modeled in the GEONS filter (3) Solution 3 in which the filter covariance was increased once for about 1 hour around the third maneuver for all satellites, and (4) Solution 4 in which the filter covariance was increased twice, around the third maneuver, as was done in the case of Solution 3, and around the last maneuver for about 2 minutes. The solution error characteristics were similar for all satellites.

Fig. 4 shows the relative position errors for Satellite 3 for each of the solutions. The Solution 1 results are provided for comparison purposes. The Solution 2 results are the worst as expected; however, the GEONS filter was able to reconverge without any covariance inflation commands after the first perigee in the post-maneuver region, when many GPS PR measurements were available. In Solution 2, the absolute position errors (not shown) reach a maximum of 700 m, which is well below the MMS science navigation accuracy

requirements of 100 km. The relative errors meet the requirement of 1 percent of the separation most of the time except for a brief period of time when the relative errors go beyond 100 m (1% of the separation distance near the apogee) to a maximum of 120 m. In Solution 2, the actual relative errors are much larger than the filter root-sum-variance as shown in Fig. 4.
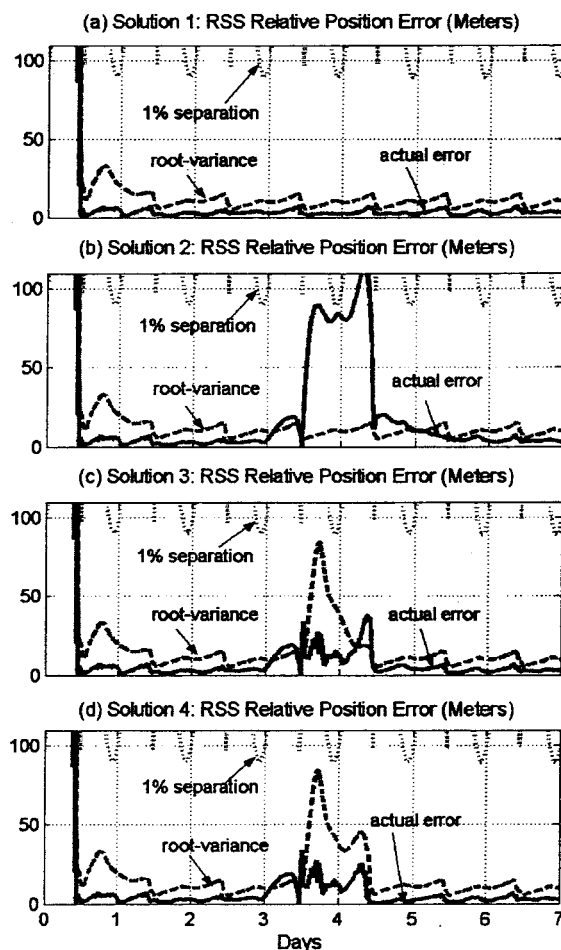

Fig. 4. Satellite 3 Relative Position Errors

The covariance around the maneuver time was increased in Solutions 3 and 4. In Solution 3, the covariance was increased over a time span of 65 minutes centered about the perigee to reduce the impact caused by three delta-V's applied around the perigee. With Solution 4, in addition to the covariance increase around the perigee, the covariance was also increased for 2 minutes to handle the last maneuver near the apogee. The time variations of the actual relative errors and root-variances for these solutions are also shown in Fig. 4. These two solutions give similar results, except that the relative covariance behavior is somewhat better with Solution 4. The absolute and relative errors of Solutions 3 and 4 are much smaller than for Solution 2.

A Monte Carlo simulation was performed based on the input data used for Solution 4. The simulation was performed only for a definitive time span of 7 days. The 25-solution Monte Carlo statistics for the steady-state

region of the definitive arc are compared in Fig. 5 with single solution results. The label 'mc-sol4' in this figure represents the Monte Carlo results, and 'sol1', 'sol2', etc. represent the four solutions discussed earlier. The Monte Carlo results are similar to the corresponding single solution results in terms of the RMS errors. The maximum errors are about 40% larger. From these results, it can be concluded that, in the presence of small formation maintenance maneuvers, the GEONS filter can provide, without any sophisticated thrust modeling, navigation solutions that will meet the MMS science absolute and relative accuracy requirements.
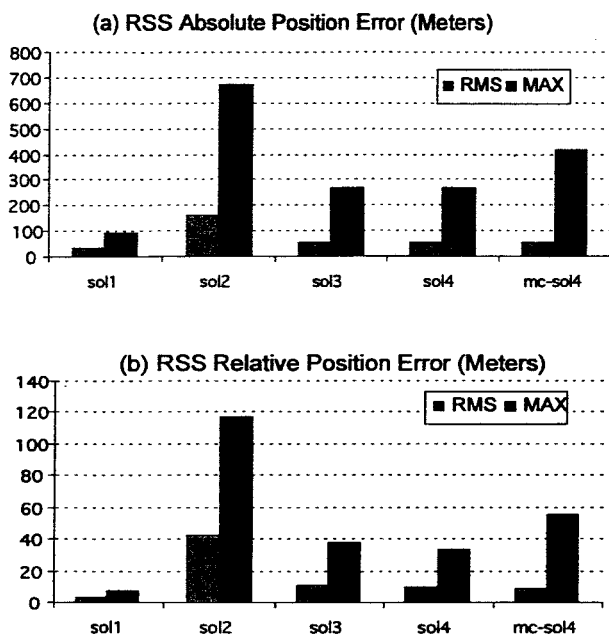

(a) RSS Absolute Position Error (Meters)

(b) RSS Relative Position Error (Meters)

Fig. 5. Satellite 3 Steady State RMS and Maximum Position Errors

2. SUMMARY OF MMS PHASE 2 NAVIGATION SOLUTIONS

GPS PR, two-way cross-link range, and GS two-way Doppler and range measurements were simulated for the MMS Phase 2 formation using the models discussed in Section 2 and processed using GEONS. This section summarizes results from this analysis.

2.1 Solutions Obtained Using GPS PR, and GPS PR plus Cross-link Range Measurements

Table 4 provides the steady-state position error statistics for solutions obtained using only GPS PR and GPS PR and cross-link range measurements. Error characteristics of the other remote satellites are similar to those for Satellite 4. Both solutions were obtained using a GPS PR measurement SD of 100 m. In the solution where GPS PR and cross-link range measurements were processed together, the cross-link range measurements

were introduced 2 days after the solution epoch (after processing GPS PR measurements near the first perigee passage) with a cross-link range measurement SD of 100 m.

Table 4 indicates that the absolute position errors for both solutions are similar and well below the 100-km position accuracy requirement. During the long periods without GPS measurements, the absolute position errors grow to about 12 km until the next perigee approach where processing a large number of GPS measurements reduces the absolute position errors to below 1 km. Without cross-link range, the relative position accuracy requirements are not met for about 1 day prior to perigee. With the cross-link range measurements included in the solutions, the relative position errors are much smaller than 1% of the inter-satellite separations.

Table 4. Position Errors for Solutions Using GPS PR and GPS PR and Cross-Link Range

Solution		Absolute Position Error (Km)		Relative Position Error (m)	
		RMS	Max	RMS	Max
GPS PR	Sat1	3.2	11.8		
	Sat4	3.2	11.6	127.5	411.1
GPS PR, Cross-link range	Sat1	3.2	11.5		
	Sat4	3.2	11.5	13.7	22.4

Thus, in terms of absolute and relative position accuracy, solutions obtained using only GPS PR (with 28 dB-Hz signal acquisition threshold) are not acceptable for the MMS Phase 2 mission navigation support; whereas, solutions obtained using GPS PR and cross-link range measurements are acceptable.

2.2 Solutions Obtained Using GS and Cross-link Range Measurements

Table 5 summarizes the steady state solutions obtained using GS and cross-link range measurements. Solution 1, which was obtained using only GS Doppler measurements, gives smaller absolute position errors (less than 3 km maximum) than the GPS PR solutions in Table 4, but does not meet the relative position accuracy requirements. Solution 2, which was obtained using GS Doppler and cross-link range measurements, provides acceptable absolute and relative solutions when the cross-link range measurements were introduced with a measurement SD of 200 m after processing approximately 6 days of GS Doppler data. Fig. 6 shows the absolute and relative errors of Solution 2 for Satellites 1 and 4. The introduction of cross-link range measurements reduced the relative position errors to below 100 m throughout the steady-state region (and below 1% of inter-satellite separation distance throughout).

Solutions 3 and 4 in Table 5 indicate that the addition of GS range measurements improves both the absolute and relative solutions. Solution 3, which was obtained using only GS range and Doppler measurements, can meet the absolute position accuracy goal, but the relative position accuracy requirements are not met all the time. Solution 4, in which cross-link range measurements were introduced 2 days after the solution epoch using a cross-link range SD of 100 m, satisfies both the MMS absolute and relative position accuracy requirements.

Table 5. Position Errors Using GS and GS plus Cross-link Measurements

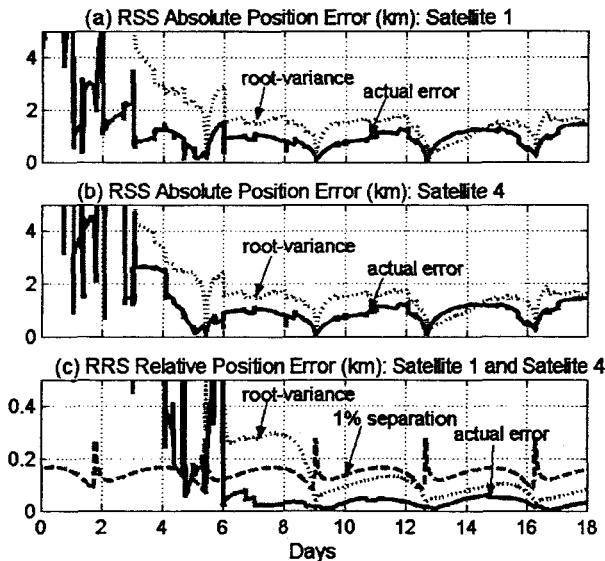| Solution		Absolute Position Error (Km)		Relative Position Error (m)	
		RMS	Max	RMS	Max
GS Doppler	Sat1	1.5	2.5		
	Sat4	0.9	1.6	728	1305
GS Doppler, Cross-link Range	Sat1	1.0	1.4		
	Sat4	0.9	1.4	33.1	61.6
GS Doppler, GS Range	Sat1	0.4	0.9		
	Sat4	0.3	0.8	159.9	328
GS Doppler, GS Range, Cross-link Range	Sat1	0.5	1.0		
	Sat4	0.5	1.0	9.2	15.4

Fig. 6. Solutions Using GS Doppler and Cross-link Range

3. CONCLUSIONS

Table 6 compares the expected maximum steady-state position errors for the OD scenarios evaluated in this paper. All three OD scenarios for MMS Phase 1 provide acceptable absolute and relative position accuracies for the science requirements. Other acceptable OD scenarios for MMS Phase 1 that were previously

identified in [3] include (1) using only GPS PR measurements and (2) using GS two-way Doppler and two-way cross-link range measurements.

For MMS Phase 2 navigation, only the three scenarios that include cross-link range measurements provide acceptable solutions. The GPS PR plus cross-link scenario can be used onboard when near-real-time navigation solutions and GPS receiver clock solutions are required. The absolute errors using GPS PR are significantly larger than for Phase 1 because of the reduced GPS visibility in Phase 2. The GS plus cross-link scenarios are appropriate for ground OD support.

Table 6. Summary of Steady-State Position Errors

	Maximum Absolute Error (km)	Maximum Relative Error (m)
Science Requirements	100	100 at apogee
Phase 1 Scenarios:		
GPS PR, cross-link range	0.3	10
Single-satellite solutions using GPS PR for remote, GPS PR and cross-link range for local satellite	0.2	130*
Small formation maintenance maneuver solutions using GPS PR and cross-link range	0.4	60
Phase 2 Scenarios:		
GPS PR	12	510
GPS PR, cross-link range	12	23
GS Doppler	3	1300
GS Doppler, cross-link range	2	60
GS range and Doppler	1	350
GS range and Doppler, cross-link range	1	25
*<75 m near apogee, maximum occurs near perigee		

Additional studies on MMS mission navigation strategies that are currently in progress or planned include: MMS Phase 4 navigation analyses, analyses using larger formation maintenance and resizing maneuvers, and Phase 3 (double-lunar swing-by phase) navigation approaches.

References

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